

Title: The Four Color Theorem: Generalizations and Faster Algorithms

Abstract: The celebrated Four Color Theorem (4CT) was conjectured by Francis Guthrie in 1852, and remained open for more than 100 years, until Appel and Haken found a proof in 1976. The proof is computer-assisted, and a simplified version was found by Robertson et al. in 1997. It has been known since 1880 (by Tait) that the 4CT is equivalent to stating that every 2-connected cubic planar graph is 3-edge-colorable. In our ongoing project, we aim to leverage today's computing power to extend the computer-assisted proof (and its computer-free extension) of the 4CT, yielding significantly stronger results and a much faster algorithm. Topics include:

- 1) Three-edge cubic graphs on a surface: Here, we completely characterize all non-three-edge colorable cubic graphs (i.e., snarks) that can be embedded in a projective plane or in a torus (while the 4CT has no extension to other surfaces).
- 2) A much faster algorithm to 4-color planar graphs, namely: We can 4-color in $O(n \log n)$ time, improving the 30-year-old algorithm by Robertson et al.
- 3) 4-coloring almost all vertices of graphs on a surface (including Albretson's conjecture from 1980).

Biography: Ken-ichi Kawarabayashi is a professor at the National Institute of Informatics and the University of Tokyo. He is working on graph algorithms and graph theory (discrete math), and he has published more than 60 papers in STOC/FOCS/SODA, which are the best algorithm conferences. He has also published more than 50 papers in J. Combin. Theory Ser. B, which is the best journal in discrete mathematics. He received the SADA'13 best paper award, the Humboldt Research Award in 2024, and the Fulkerson Prize in 2021, which is the highest prize in discrete mathematics and graph algorithms. He is also an ACM fellow in 2025. He is the general co-chair of SODA'12, which, for the first time, took place outside North America. He gave many plenary talks, including at ICALP'15 and at SODA'26 (the best conference in algorithms). He is currently co-editor-in-chief of Graphs and Combinatorics and an editor of Theoretical Computer Science, Algorithmica, and J. Graph Theory.